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COMMENT

On the phase transitions in superconductors with quenched impurities

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Abstract. The question whether the Halperin–Lubensky–Ma result for a fluctuation-induced weakly first-order phase transition in superconductors holds in the presence of quenched impurities is considered. The renormalisation-group recursion relations have a new stable fixed point for $1 < n \leq 366$, which describes a real critical behaviour in the range $2 < D_c(n) < d < 4$ of space dimensionalities d ($n/2$ is the number of components of the complex order parameter). Some features of the new fixed point are discussed. The critical exponents are presented for $1 < n \leq 366$.

The possibility of a weakly first-order phase transition (FOT) was demonstrated for the first time on the basis of the Ginzburg–Landau (GL) model of superconductors (Halperin *et al* 1974). A comprehensive discussion of this fluctuation induced effect is presented by Chen *et al* (1978). The effect has been established for the model describing the nematic–smetic-A transition in liquid crystals (Halperin and Lubensky 1974, Lubensky and Chen 1978), abelian Higgs models (Lawrie 1982a), and models with two order parameters (Tonchev and Uzunov 1981a, b). It is a consequence of the interaction between the fluctuation order parameter and a gauge field, such as the vector potential $\boldsymbol{A}(\boldsymbol{x})$ in the GL free energy. If the gauge field is neglected, the phase transition is of second order. Besides, the studies in this field are essentially used when a successful gauge theory of spin glasses is looked for (Hertz 1978).

Recently, a three-dimensional lattice superconductor model was studied by means of Monte Carlo simulations (Dasgupta and Halperin 1981). The FOT was not found. Besides, Lawrie (1982b) has demonstrated that 'there is a value of the electric charge e above which the argument for the FOT fails'. These new results, supported by experiments in liquid crystals, make vulnerable the statement that the fluctuation-induced FOT takes place for any value of the parameters of the theory. Experiments in ordinary superconductors cannot help for the solution of the problem as the size of the FOT was found to be experimentally undetectable (Halperin *et al* 1974) with the experimental techniques available at present. There is, in principle, a possibility for the FOT to be observed in complicated superconductors with bicritical and tetra-critical points, where a great enlargement of the critical region has been predicted (Hornreich and Schuster 1979, Tonchev and Uzunov 1981a, b). This may turn out to be a real way for an experimental investigation of the FOT in superconductors.

On the other hand, when trying to plot experiments against theory, one has to remember that in real systems inhomogeneities of several types always exist, some of them being relevant to the scaling properties (Lubensky 1975, Ma 1976). Viewing

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the problems in pure gauge-invariant systems, it is interesting to understand the way in which the inhomogeneities modify the theoretical predictions for a FOT. Contrary to the effect of the vector potential $\boldsymbol{A}(\boldsymbol{x})$, the quenched impurities act to smear the transition. The renormalisation-group (RG) treatment by means of the Callan–Symanzik equations reveals a spectacular competition between the effect of the vector potential and the quenched-impurity influence (Boyanovsky and Cardy 1982).

In the present paper we investigate the same influence on the phase transition in a superconducting state, using the Wilson–Fisher method of the finite recursion relations (see Ma 1976). Our results are complementary to those of Boyanovsky and Cardy (1982).

The GL model $\mathscr{H}=(-H/k_BT)$ in the presence of quenched impurities is given by

$$\mathscr{H}=-\int \mathrm{d}\boldsymbol{x}\,[a|\psi|^2+\gamma|(\nabla-\mathrm{i}q_0\boldsymbol{A})\psi|^2+\tfrac{1}{2}B|\psi|^4+(8\pi\mu)^{-1}(\mathrm{rot}\,\boldsymbol{A})^2], \tag{1}$$

where $q_0=(2e/\hbar c)$, $\mathrm{div}\,\boldsymbol{A}(x)=0$ and $\psi(\boldsymbol{x})$ is generalised to an $(n/2)$-component complex order parameter. The parameters $\gamma=\hbar^2/2m$, b, and $a=a'(T-T_c)T_c+\varphi(\boldsymbol{x})$ are spatially dependent. However, for calculations to order ε^1, $\varepsilon=4-d$ (d is the dimensionality of space), the only one relevant to the RG analysis spatial dependence is that represented by the random function $\varphi(x)$ with a gaussian distribution

$$\langle\varphi(\boldsymbol{k})\varphi(\boldsymbol{k}')\rangle=\Delta(\boldsymbol{k})\delta_{-\boldsymbol{k},\boldsymbol{k}'} \tag{2}$$

where $\Delta(k)$ is a non-negative function of the momentum $\boldsymbol{k}$, $0<k<\Lambda(k\equiv|\boldsymbol{k}|)$. We examine the momentum-independent ($\Delta(k)\approx\Delta$) large-distance behaviour of the impurity correlation function $\langle|\varphi(\boldsymbol{k})|^2\rangle$ since the more general even-power k-dependence (Larkin and Ovchinnikov 1971) also turns out to be irrelevant to the scaling analysis despite of the presence of the vector potential $\boldsymbol{A}(\boldsymbol{x})$ in the model (1). We shall apply the well known technique (Lubensky 1975, Ma 1976) for studying the impurity effects without the use of the replica trick (Edwards and Anderson 1975). We choose units in which $\Lambda=K_B=T_c=1$.

The recursion relations to order ε^1 are

$$a'=s^{2-\eta_\psi}\left[a+\frac{1}{8\pi^2}\left(\frac{n+2}{2}B-\Delta\right)\left(\frac{1-s^{-2}}{2\gamma}-\frac{a}{\gamma^2}\ln s\right)+\frac{3}{4\pi}(1-s^{-2})\gamma q_0^2\mu\right], \tag{3a}$$

$$B'=s^{\varepsilon-2\eta_\psi}\{B+(B/8\pi^2\gamma^2)[6\Delta-\tfrac{1}{2}(n+8)B]\ln s-12\gamma^2(q_0^2\mu)^2\ln s\}, \tag{3b}$$

$$\Delta'=s^{\varepsilon-2\eta_\psi}\{\Delta+(\Delta/8\pi^2\gamma^2)[4\Delta-(n+2)B]\ln s\}, \tag{3c}$$

$$\gamma'=s^{-\eta_\psi}\gamma[1-(3/2\pi)(q_0^2\mu)\ln s], \tag{3d}$$

$$(1/\mu')=s^{-\eta_A}\mu^{-1}[1+(n/12\pi)(q_0^2\mu)\ln s], \tag{3e}$$

$$q_0'=s^{(\varepsilon-\eta_A)/2}q_0, \tag{3f}$$

where $s>1$ is the RG rescaling factor, η_ψ and η_A are the anomalous dimensionalities (Fisher's exponents) of the fields ψ and $\boldsymbol{A}$, respectively. The recursions (3) are a direct generalisation of those for pure ($\Delta=0$) superconductors (Halperin *et al* 1974), and for the φ^4 theory accounting for quenched impurities (Lubensky 1975). This fact is a consequence of the gauge properties of the vector potential and is valid for other gauge models as well.

A gaussian (unstable for $\varepsilon>0$) fixed point (FP) of the relations (3) always exists. Searching for stable FPs, we find the same values $\eta_A=\varepsilon$ and $\eta_\psi=-18\varepsilon/n$ as in the pure case. Introducing new parameters $t = q_0^2\mu$, $r = a/\gamma$, $u = b/8\pi^2\gamma^2$ and $\zeta = \Delta/2\pi^2\gamma^2$, we obtain for $n > 1$ FPS with coordinates

$$t^* = 12\pi\varepsilon/n, \tag{4a}$$

$$r_\pm^* = \tfrac{1}{8}\zeta_\pm^* - \tfrac{1}{4}(n+2)u_\pm^*, \tag{4b}$$

$$u_\pm^* = [(n+36)/4n(n-1)][1 \pm f]\varepsilon, \tag{4c}$$

$$\zeta_\pm^* = [(n+36)/4n(n-1)][3(2-n) \pm (n+2)f]\varepsilon, \tag{4d}$$

where

$$f(n) = [1 + 3456(n-1)/(n+36)^2]^{1/2}.$$

Only the FP with r_+^*, u_+^* and ζ_+^* has to be considered as a physical FP (Lubensky 1975), because ζ_-^* is negative for any $n > 1$, whereas $\zeta_+^* > 0$ in the range $1 < n \leqslant 366 = n_c$ (n integer). The critical value $n_c = 366$ can be obtained from (4) after a simple computer calculation. Hence, the presence of the parameter $\Delta \geqslant 0$ leads to the emergence of a new FP for a range of values of n where no real FPS were 'available' in the pure superconductor (Halperin *et al* 1974). In the last case, the equation for $u_\pm^*$ has complex solutions for $n < 365.9$. The FP (4), its interesting properties, as well as the critical exponents for $n = 2$, have already been obtained by Boyanovsky and Cardy (1982).

The critical exponents in the range $1 < n \leqslant 366$ are

$$\nu = \tfrac{1}{2} - (\varepsilon/4n)\{18 + [(n+36)/16(n-1)][2 - 5n - (n+2)f]\}, \tag{5}$$

$$y_\pm = X(\varepsilon, n) \pm \mathrm{i}\,Y(\varepsilon, n), \tag{6}$$

with

$$X(\varepsilon, n) = \left(1 + \frac{36}{n}\right)\varepsilon + \frac{n+36}{16n(n-1)}[22 - 25n + 3(n-2)f], \tag{7a}$$

$$Y(\varepsilon, n) = \frac{n+36}{16n(n-1)}\,\varepsilon\{24(n+2)(1+f)[6 - 3n + (n+2)f] - [18 - 3n + (n+14)f]^2\}^{1/2} \tag{7b}$$

where $y_{+,-} \equiv y_{u,\zeta}$, and the exponents y_τ, $\tau \equiv (t, r, u, \zeta)$, are defined by the relation $\delta\tau' = s^{y_\tau}\delta\tau$. As $X(\varepsilon, n)$ is negative for $\varepsilon > 0$ the FP is stable with respect to u- and ζ-perturbations. The imaginary part $Y(\varepsilon, n) \neq 0$ leads to spiral RG trajectories to the FP which has accordingly been distinguished as a focus of the RG transformation. The critical behaviour corresponding to the focal FP will have a physical sense if the critical exponents ν, δ and β are positive. For ν this is seen from equation (5). Using a well known scaling relation (Ma 1976), one can check that $\delta > 0$ too. The exponent β is a given by the relation

$$\beta = \tfrac{1}{2}\nu(d - 2 + \eta_\psi).$$

Using $\nu > 0$, $\eta_\psi = -18\varepsilon/n$ and the requirement $\beta > 0$, we obtain the condition

$$d > D_c(n) = 2(n+36)/(n+18), \tag{9}$$

which is a restriction for the dimensionality of space. If $d \leqslant D_c(n)$ the new critical behaviour should not be considered as real. In this case, using the non-RG arguments for a FOT in pure superconductors (Halperin *et al* 1974), we may conclude that the FOT manifests itself again despite the presence of quenched impurities. For $n = 2$ (the case of a superconductor), $D_c(2) = 3.8$. A conclusion for a FOT can be verified, if one repeats the self-consistent calculations of Halperin *et al* (1974) with regard to the present case. This alternative is the most appealing one for three-dimensional systems. It must be stressed that the above-mentioned self-consistent calculation is, in fact, insensitive to the impurities. Thus, a new unconventional self-consistent method is needed for the resolute verification of the FOT in impure systems.

The next remark concerns the ε^2 corrections. We have not a fair chance of success if we try to analyse the recursion relations in $O(\varepsilon^2)$. The recursion relations of order ε^2 are not analytically tractable when the impurities are included, even within the framework of the standard φ^4-theory. The situation is the same with the analytic study of the pure superconductor. Besides, there are no physical arguments that there will be a qualitative disagreement with the present calculation.

From (5)–(7) one can check that the well known relation

$$2\nu^{-1} - d = y_\zeta \tag{10}$$

does not hold here. The reason is that, owing to the vector potential $\boldsymbol{A}(\boldsymbol{x})$, the model (1) does not possess spherical symmetry. If we reformulate the RG transformation, introducing a rescaling for the random function via the relation $\varphi(s\boldsymbol{k}') = s^{y_\varphi}\varphi(\boldsymbol{k})$ and go through the derivation of the relation (10) (Ma 1976), we get

$$2y_\varphi - d = y_\zeta. \tag{11}$$

The new relation (11), which holds for any system, gives us information about the scaling properties of the function $\varphi(\boldsymbol{k})$.

Now we end with a discussion on a property of the focal FP (4). Although the irrelevant exponents y_u and y_ζ are complex, it will be wrong to conclude that the free energy F of the system is a non-analytic function of the variations $\delta u = u - u_\mp^*$ and $\delta\zeta = \zeta - \zeta_\mp^*$ near the FP. If the free energy is a non-analytic function, it will have (at least) poles in the variables $\delta u' = s^{y_u}$, δu and $\delta\zeta' = s^{y_\zeta}\delta\zeta$ (see Wegner 1976). Then, choosing $s = \xi = (T - T_c)^{-\nu}$, we find that the parameters u and ζ act as relevant ones. In fact, the free energy can be explicitly calculated as a function of the renormalised parameters, and in this way a realisation of Wegner's expansion in the powers of $\delta\tau$ is to be obtained (Wegner 1976). On the other hand, $F(\tau)$ must be a real function of δu and $\delta\zeta$. With the last condition in mind, a new property of the local FP can be derived. It is also valid for other models with complex exponents (Uzunov 1983). The result from the calculation of $F(\tau)$ to $O(\tau')$ is

$$F(\tau) = F_0(\tau^*) + F_1(\delta r, \delta t) + F_2(\delta u, \delta\zeta) + O(\varepsilon, \delta\tau)$$

with

$$F_2 = \tfrac{1}{2}\pi^2 n\gamma^2[2(n+2)\delta u' - \delta\zeta' - \delta\zeta']J^2(0),$$

where

$$J(r) = \int \frac{d^d k}{(2\pi)^d}\frac{1}{\gamma k^2 + r}, \qquad 0 < k < 1.$$

Using the linearised transformations $\delta u' = s^{y_u}\delta u$ and $\delta\zeta' = s^{y_\zeta}\delta\zeta$ as well as $y_{u,\zeta}$ from (6)–(7), the condition that F_2 must be a real function of δu and $\delta\zeta$ yields

$$2(n+2)\delta u + \delta\zeta = 0 + O(\varepsilon\,\delta\tau) \tag{12}$$

which is a restriction on the possible variations of δu and $\delta\zeta$ around the focal FP. Also, equation (12) shows that the RG trajectories to the FP have one and the same tangent.

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